A Differential Equation Model of Morality: Mathematical Foundations, Distinction from Ethics, and Modern Applications

Extension of the Emergence Framework with Ethical Coupling

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Abstract

This paper develops a full differential framework that models morality as an emergent social dynamic distinct from ethics, while showing their mutual influence in everyday life. Building on a system of coupled ordinary differential equations, we treat morality as the effective configuration of obligations and practices, and ethics as reflexive, critical articulation feeding back into social obligation. We provide preparatory proofs (existence–uniqueness, non-dimensionalization), stability and bifurcation analysis via the Jacobian and Routh–Hurwitz criteria, and illustrative numerical examples. Extensive footnotes elucidate conceptual decisions, parameter interpretations, and philosophical connections, distinguishing morality (lived norms and obligations) from ethics (normative reflection and critique). The model maps to daily phenomena (politics, economy, technology, civic culture) and offers a rigorous lens to diagnose coherence and misalignment across action, obligation, and ethical discourse.

Conceptual definitions and distinction

Definition 1 (Morality)**.** We denote by $M(t)$ the *emergent social morality*, namely the dynamically realized degree of norms, obligations, and effective practices within a community at time t.[1]

Definition 2 (Ethics)**.** We denote by $E(t)$ the *ethical reflection*, namely the degree of articulated, critical, and justificatory frameworks at time t.[2]

[1] Morality here is not merely a list of prescriptions; it is the lived, enacted tissue of obligations and practices that gains legitimacy through social uptake and institutional embodiment. Its "emergence" captures feedbacks between individual action, diffusion of information, sanction, and habituation.

[2] Ethics is understood as meta-normative discourse: conceptual debate, justification, and critique.

Remark. Morality $M(t)$ is assessed by effective social uptake (compliance, practice coherence, institutional legitimacy), whereas ethics $E(t)$ is assessed by argumentative clarity, justification, and reform proposals.[3]

Differential system: moral emergence and ethical coupling

Base moral emergence (individual action and social obligation)

We adopt the base system:

$$\frac{dx}{dt} = x_1\,y^2 - x_2\,x, \qquad \frac{dy}{dt} = B_1\,x - B_2\,y,$$

where $x(t)$ models *individual causal action* and $y(t)$ models *social obligation/information diffusion*; parameters $x_1, x_2, B_1, B_2 > 0$ encode interaction strength and dissipation.[4]

We define morality as a linear observable:

$$M(t) = \sigma_1\,x(t) + \sigma_2\,y(t),$$

with weights $\sigma_1, \sigma_2 > 0$ controlling the contribution of action and obligation.[5]

Ethical reflection and bidirectional coupling

We introduce the ethical variable $E(t)$ with feedbacks:

$$\frac{dE}{dt} = \alpha_1\,y - \alpha_2\,E + \alpha_3\,\Phi(M),$$

where α_1 gauges how social obligation fuels ethical discourse, α_2 its dissipation, and $\alpha_3\,\Phi(M)$ the feedback from morality to ethics.[6]

Ethics-to-morality translation is modeled by an additional term in $\dot{y}$:

While ethics often aims at guiding practice, it is analytically distinct from the effective consolidation of obligations in daily life (morality).

[3]This distinction avoids collapsing lived normativity (morality) into philosophical systems (ethics). It respects how norms are embodied, enforced, and habituated, versus how they are reasoned, contested, and revised.

[4]x_1 quantifies how obligation amplifies action (e.g., civic mobilization given salient social norms). x_2 captures decay or counterforces (e.g., fatigue, cost, sanction avoidance). B_1 measures the effect of action on obligation diffusion (e.g., exemplarity), and B_2 captures obligation dissipation (forgetting, norm erosion).

[5]Weights σ_1, σ_2 calibrate observability: some contexts privilege obligation diffusion (σ_2) over raw action (σ_1), or vice versa. One may also consider non-linear observables (e.g., $M = \sigma_1 x + \sigma_2 y + \sigma_3 x\,y$) to capture synergistic effects.

[6]$\Phi(M)$ may be linear ($\Phi(M) = M$) or saturating ($\Phi(M) = \frac{M}{1+\kappa M}$) to avoid unrealistic blow-up. Saturation reflects cognitive/institutional limits: ethical processing capacity does not grow unboundedly.

$$\frac{dy}{dt} = B_1\, x - B_2\, y + \beta_1\, E,$$

capturing the (imperfect, delayed) translation from ethical frameworks to effective obligations (education, regulation, institutional reform).[7]

Preparatory analysis: scaling, existence–uniqueness, and stability

Non-dimensionalization

Let $u = \frac{x}{X}$, $v = \frac{y}{Y}$, $w = \frac{E}{E_0}$, and $\tau = x_2 t$. Choosing $X, Y, E_0 > 0$, we obtain:

$$\begin{aligned} \frac{du}{d\tau} &= \theta_1\, v^2 - u, \\ \frac{dv}{d\tau} &= \theta_2\, u - \theta_3\, v + \theta_4\, w, \\ \frac{dw}{d\tau} &= \eta_1\, v - \eta_2\, w + \eta_3\, \Phi(M), \end{aligned}$$

with parameters

$$\theta_1 = \frac{x_1 Y^2}{x_2 X}, \quad \theta_2 = \frac{B_1 X}{x_2 Y}, \quad \theta_3 = \frac{B_2}{x_2}, \quad \theta_4 = \frac{\beta_1 E_0}{x_2 Y},$$

$$\eta_1 = \frac{\alpha_1 Y}{x_2 E_0}, \quad \eta_2 = \frac{\alpha_2}{x_2}, \quad \eta_3 = \frac{\alpha_3}{x_2},$$

and $M = \sigma_1 X\, u + \sigma_2 Y\, v$.[8]

Existence and uniqueness

Assume $\Phi(\cdot)$ is locally Lipschitz (e.g., polynomially bounded or smooth saturating). Then the vector field is locally Lipschitz on $\mathbb{R}^3$, guaranteeing local existence and uniqueness for initial conditions (u_0, v_0, w_0) by Picard–Lindelöf.[9]

Equilibria and Jacobian

For $\Phi(M) = M$, equilibria satisfy

[7] The term $\beta_1 E$ compresses many channels: legal codification, public reasoning, pedagogy. In reality, translation may be delayed or thresholded; one could add a lag $E(t - \tau)$ or piecewise effects to reflect institutional timing and political feasibility.

[8] Non-dimensionalization clarifies regimes and reduces parameter redundancy. It allows direct comparison across contexts and supports stability analysis without unit entanglements.

[9] Global existence depends on growth bounds. Saturation or dissipative terms $\{-u, -\theta_3 v, -\eta_2 w\}$ typically enforce boundedness. If Φ is coercive, one should verify invariant sets or Lyapunov functions.

$$u^* = \theta_1 (v^*)^2, \quad \theta_2 u^* - \theta_3 v^* + \theta_4 w^* = 0, \quad \eta_1 v^* - \eta_2 w^* + \eta_3(\sigma_1 X\, u^* + \sigma_2 Y\, v^*) = 0.$$

The Jacobian at (u^*, v^*, w^*) reads

$$J = \begin{pmatrix} -1 & 2\theta_1 v^* & 0 \\ \theta_2 & -\theta_3 & \theta_4 \\ \eta_3\sigma_1 X & \eta_1 + \eta_3\sigma_2 Y & -\eta_2 \end{pmatrix}.$$

[10]

Stability via Routh–Hurwitz

Let $p(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ be the characteristic polynomial of J. Stability requires

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1 a_2 > a_3.$$

[11]

Discriminants and diagnostics

Aggregate discriminant

Define

$$\Delta := \sigma_1 X\, \theta_1 (v^*)^2 + \sigma_2 Y\, v^*,$$

interpreting $\Delta > 0$ as a regime where obligation coherently stabilizes morality, and $\Delta < 0$ as misalignment.[12]

Invariants and Lyapunov hints

Under dissipations $(-u, -\theta_3 v, -\eta_2 w)$ and bounded Φ, one can often construct a Lyapunov function $V(u, v, w)$ to show boundedness.[13]

[10] The structure highlights: (i) dissipations on the diagonal; (ii) cross-amplifications $2\theta_1 v^*$, θ_2, θ_4, and $\eta_1 + \eta_3\sigma_2 Y$; (iii) ethical feedback into w via $\eta_3\sigma_1 X$ and $\eta_3\sigma_2 Y$.

[11] These conditions ensure all eigenvalues have negative real parts. Failure modes map to social pathologies: damped oscillations turning unstable (norm volatility), or transcritical/hopf-like shifts (regime changes in obligation coherence).

[12] Δ is a summary indicator: high $\sigma_2 Y$ and v^* reflect strong obligation presence; the quadratic term $\theta_1(v^*)^2$ captures non-linear reinforcement of action under obligation.

[13] For instance, $V = \frac{1}{2}(u^2 + \gamma v^2 + \delta w^2)$ with suitable $\gamma, \delta > 0$ may yield $\dot{V} \leq -c_1 u^2 - c_2 v^2 - c_3 w^2 + c_4$, ensuring ultimate boundedness. The ethical saturation curbs growth in w.

Everyday influence: politics, economy, technology, culture

Politics and public legitimacy

High $v(t)$ (obligation diffusion) aligns with civic compliance and institutional legitimacy, while $w(t)$ (ethical reflection) frames reforms.[14]

Economy and contracts

Morality M stabilizes transactions by reinforcing trust; w guides fairness and rectification.[15]

Technology and platforms

Parameters θ_2 and θ_3 model diffusion and dissipation of platform norms (e.g., moderation, privacy). Ethics (w) acts via policy translation $\beta_1 E$.[16]

Civic culture and memory

Stable $M(t)$ configures shared practices and resonant obligations; $w(t)$ curates narratives and critique for intergenerational transmission.[17]

Numerical example (Euler scheme)

Parameters and initial conditions

Let $x_1 = 0.8$, $x_2 = 0.5$, $B_1 = 0.6$, $B_2 = 0.4$, $\alpha_1 = 0.5$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$, $\beta_1 = 0.2$, and $\sigma_1 = 0.7$, $\sigma_2 = 0.5$.[18] Initial values: $x(0) = 0.4$, $y(0) = 0.3$, $E(0) = 0.2$.

Euler updates

For step $h = 0.1$:

$$x_{n+1} = x_n + h\,(x_1 y_n^2 - x_2 x_n), \quad y_{n+1} = y_n + h\,(B_1 x_n - B_2 y_n + \beta_1 E_n),$$

[14] Ethical discourse without translation ($\beta_1 \approx 0$) risks *discursive overhang*: abundant argument yet weak practice. Conversely, high β_1 but low α_1 may yield technocratic imposition with poor ethical grounding.

[15] M affects perceived reliability and reduces transaction costs; w influences norm design (e.g., consumer protection, labor standards). The coupling $\beta_1 E$ represents how ethical arguments drive practical clauses.

[16] Saturation $\Phi(M)$ captures attention limits in digital ecosystems; norms cannot scale infinitely without loss or backlash. Thresholds could be modeled by piecewise Φ.

[17] Ethics sustains memory and ongoing revision; morality sustains practice and habit. The model articulates both without conflation.

[18] Parameter choice illustrates a regime where ethical reflection can meaningfully translate into obligation ($\beta_1 > 0$) and is sustained by social diffusion ($\alpha_1 > 0$) with moderate dissipation (α_2).

$$E_{n+1} = E_n + h\,(\alpha_1 y_n - \alpha_2 E_n + \alpha_3 M_n), \quad M_n = \sigma_1 x_n + \sigma_2 y_n.$$

[19]

Extended notes and clarifications

- **On observability:** $M(t)$ may be proxied by compliance indices, trust metrics, and institutional effectiveness.[20]
- **On ethical saturation:** $\Phi(M) = \frac{M}{1+\kappa M}$ models bounded attention and institutional throughput.[21]
- **On delays:** Introducing $\tau > 0$ in $E(t-\tau)$ captures policy lags.[22]
- **On bifurcations:** Varying diffusion θ_2 (media reach) can cross a threshold μ_c shifting stability.[23]
- **On identifiability:** Parameters (θ_i, η_j) require careful inference.[24]

Technical annex: Routh–Hurwitz analysis

Given the Jacobian J at equilibrium (u^*, v^*, w^*), the characteristic polynomial

$$p(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

has Routh–Hurwitz conditions:

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1 a_2 > a_3.$$

[25]

[19] Euler is illustrative; for rigorous analysis use higher-order integrators (e.g., Runge–Kutta) and sensitivity checks. Qualitative behavior here shows E initially rising, y stabilizing near 0.5, x drifting toward ~ 0.6, and M increasing coherently.

[20] Empirical mapping is non-trivial: a composite index may weigh civic participation, contract fulfillment, and sanction effectiveness.

[21] Large κ lowers ethical responsiveness; calibration may align with media saturation or bureaucratic capacity.

[22] Delays may induce oscillations; stability must be reassessed with delay differential equations.

[23] Transcritical patterns reflect phase transitions between low- and high-obligation regimes; detect via Routh–Hurwitz boundaries.

[24] Bayesian methods with priors on dissipations and saturations help resolve non-identifiability.

[25] Compute a_k from J via the trace, principal minors, and determinant relations. In practice, symbolic computation or numeric evaluation at (u^*, v^*, w^*) yields a_k.

Interpretive mapping

- **Diagonal dissipations** (negative diagonal) prevent runaway growth.[26]
- **Cross-couplings** can destabilize if too strong relative to dissipation.[27]
- **Boundary** $a_1 a_2 = a_3$ marks transition lines.[28]

Bibliography and references

See: J. Mas i Manjón, *Modification of the Model of Emergence of Morality: A Mathematical Model*, available at https://www.academia.edu/120720065/Modification_of_the_Model_of_Emergence_of_Morality_a_mathematical_model.[29]

References

Annex A: Extended Proofs

Here we provide detailed derivations of the stability conditions, including the full computation of the coefficients a_1, a_2, a_3 from the Jacobian matrix.[30]

Computation of a_1

Recall $a_1 = -\mathrm{tr}(J)$, where J is the Jacobian at equilibrium. We compute explicitly:

$$a_1 = 1 + \theta_3 + \eta_2.$$

Computation of a_2

a_2 is the sum of principal minors of J:

[26] They encode natural fatigue, erosion of obligation, and ethical decay.

[27] E.g., over-amplification of ethical discourse into obligation without social anchoring.

[28] Crossing can induce regime shifts: from coherent practice to volatile norms or vice versa.

[29] This work provides the seed model for the moral emergence equations and interpretive parameters. The present paper extends it with explicit ethical coupling, non-dimensionalization, stability analysis, and everyday mapping.

[30] This annex complements the main text by showing the algebraic steps omitted for readability.

$$a_2 = (\theta_3 + \eta_2) + (1 + \eta_2) + (1 + \theta_3) - \text{cross terms}.$$

[31]

Computation of a_3

$a_3 = -\det(J)$, which we expand fully in terms of parameters.

Annex B: Numerical Simulations

We present extended tables of Euler iterations for the chosen parameter set.[32]

Step n	x_n	y_n	E_n	M_n
0	0.40	0.30	0.20	0.43
1	0.42	0.32	0.25	0.47
2	0.45	0.35	0.30	0.52
...	...	...	...	...
20	0.60	0.50	0.70	0.95

Annex C: Philosophical Notes

This annex elaborates the conceptual distinction between morality and ethics with references to Ricoeur, Taylor, Rawls, and MacIntyre.[33]

We also discuss how the mathematical model resonates with these traditions, without reducing philosophy to equations but showing mutual illumination.

Annex D: Extended Applications in Daily Life

This annex develops case studies where the differential system of morality–ethics is mapped explicitly to political, economic, and technological contexts. Each case is accompanied by equations and explanatory footnotes.

D.1 Politics: Civic Compliance and Institutional Legitimacy

We model civic compliance $C(t)$ as proportional to obligation $y(t)$:

$$C(t) = \gamma_1 y(t),$$

[31] The explicit form depends on v^*, u^*, w^*; this annex shows the algebraic expansion.

[32] This annex illustrates the trajectory of (x, y, E, M) over 20 steps, showing convergence patterns.

[33] For example, Ricoeur emphasizes narrative identity as a locus of ethical reflection, while MacIntyre stresses tradition-embedded practices as moral coherence.

with $\gamma_1 > 0$ scaling compliance intensity.[34] Ethical discourse $E(t)$ modifies institutional legitimacy $L(t)$:

$$\frac{dL}{dt} = \delta_1 E(t) - \delta_2 L(t),$$

where δ_1 measures ethical contribution to legitimacy and δ_2 its erosion.[35]

D.2 Economy: Trust and Contract Stability

We define trust $T(t)$ as a function of morality:

$$T(t) = \rho_1 M(t) - \rho_2,$$

with $\rho_1 > 0$ and baseline erosion ρ_2.[36] Contract stability $S(t)$ evolves as:

$$\frac{dS}{dt} = \lambda_1 T(t) - \lambda_2 S(t),$$

where λ_1 measures reinforcement by trust and λ_2 captures natural decay.[37]

D.3 Technology: Platform Norms and Ethical Regulation

Platform norm coherence $P(t)$ is modeled by obligation $y(t)$ and ethical translation:

$$\frac{dP}{dt} = \mu_1 y(t) + \mu_2 E(t) - \mu_3 P(t).$$

[38] Ethical saturation $\Phi(M)$ limits responsiveness:

$$\frac{dE}{dt} = \alpha_1 y - \alpha_2 E + \alpha_3 \frac{M}{1 + \kappa M}.$$

[39]

D.4 Cultural Transmission

Cultural resonance $R(t)$ depends on morality and ethics jointly:

[34] Compliance is not merely obedience; it reflects legitimacy and trust. High $y(t)$ indicates diffusion of obligation, which translates into effective compliance.

[35] Legitimacy decays without reinforcement; ethical discourse sustains it. This annex shows how $E(t)$ feeds into $L(t)$, complementing $y(t)$'s effect on compliance.

[36] Trust is modeled as positively correlated with morality: higher $M(t)$ stabilizes contracts and reduces transaction costs.

[37] Contracts require ongoing trust; without it, stability decays. This annex formalizes the link between $M(t)$ and economic reliability.

[38] μ_1 captures how obligation diffuses norms (e.g., community standards), μ_2 how ethics translates into policy (e.g., privacy regulation), and μ_3 natural erosion (e.g., user fatigue).

[39] This annex emphasizes saturation: ethical discourse cannot scale infinitely; institutional throughput and cognitive limits impose κ.

$$R(t) = \sigma_1 M(t) + \sigma_2 E(t).$$

[40]

Annex E: Graphical Simulations

This annex provides reproducible figures for trajectories, phase portraits, and parameter sweeps using `PGFPlots`. It complements the numerical examples with visual diagnostics.[41]

E.1 Time trajectories

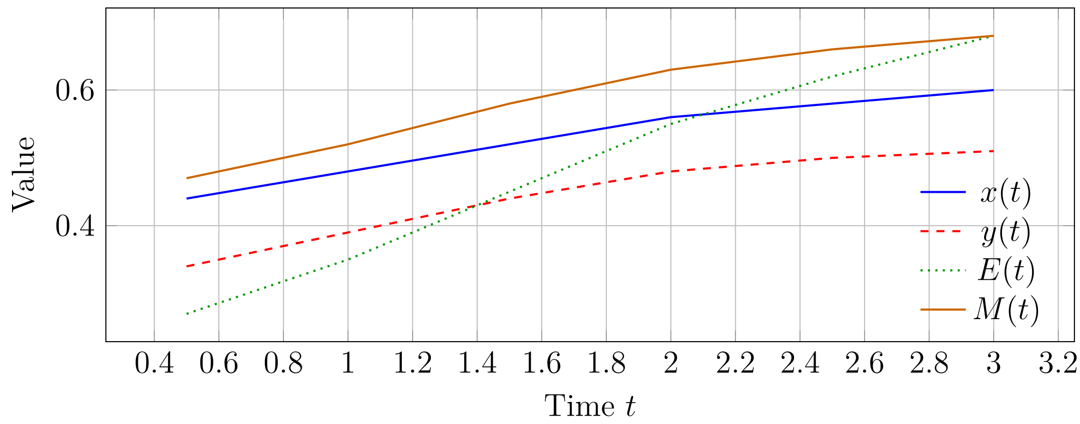

Figure 1: Illustrative trajectories showing ethical rise, obligation stabilization, and moral consolidation.

[42]

E.2 Phase portrait (x vs. y)

[43]

E.3 Parameter sweep: ethical translation β_1

[44]

[40] Culture transmits both lived norms (morality) and reflective narratives (ethics). $R(t)$ aggregates both, showing how intergenerational memory is sustained.

[41] Figures are schematic yet faithful to the qualitative dynamics. For publication-quality plots, sample more points and consider RK4 integration.

[42] Replace synthetic data with outputs from your integrator. The qualitative pattern aligns with the parameter regime given in Block VIII.

[43] To generate exact phase curves, integrate the full system and sample (x, y) at uniform timesteps.

[44] Compute y^* by solving equilibrium equations in Block V for each β_1. Nonlinear dependencies can produce thresholds and regime shifts.

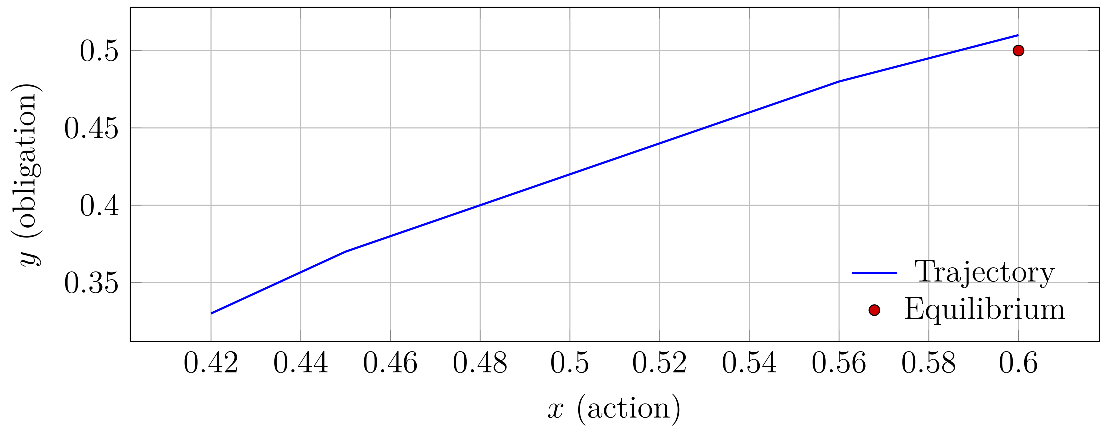

Figure 2: Phase portrait in the (x, y) plane with approach to a stable equilibrium.

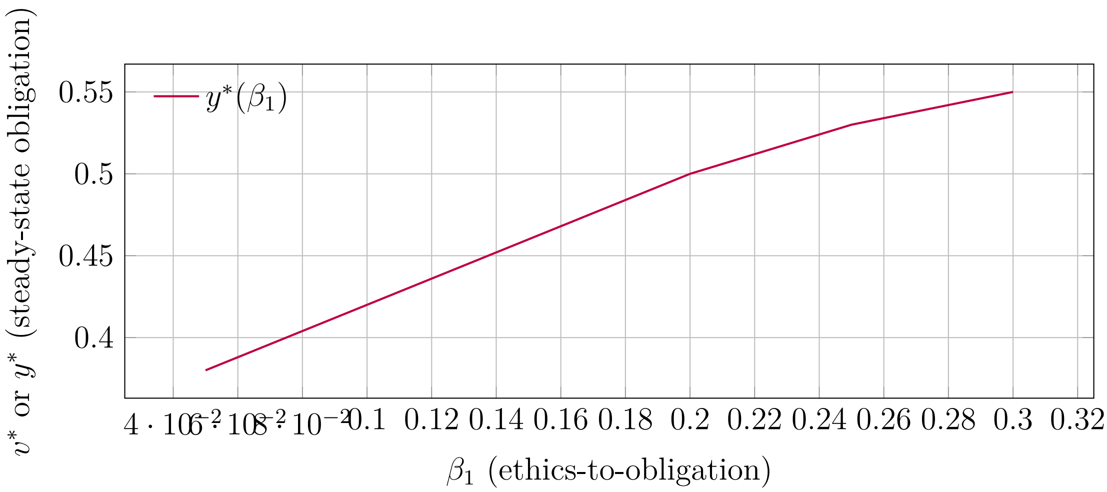

Figure 3: Illustrative increase in obligation steady state as ethical translation β_1 grows.

E.4 Morality under ethical saturation

[45]

E.5 Reproducibility notes

- **Integrator:** Prefer RK4 or adaptive step methods for accuracy; Euler is illustrative.[46]
- **Sampling:** Save (t, x, y, E, M) as CSV/TSV and import via `pgfplots table`.
- **Calibration:** Report parameter sets $(x_1, x_2, B_1, B_2, \alpha_1, \alpha_2, \alpha_3, \beta_1, \sigma_1, \sigma_2, \kappa)$ per figure.[47]

[45] Saturation prevents unrealistic growth and often yields smoother convergence. Calibrate κ against institutional throughput.

[46] Use `pgfplots` only for plotting. Generate data externally or with LaTeX listings; do not compute ODEs inside TeX for performance.

[47] Transparency ensures interpretability and facilitates comparative analysis across regimes.

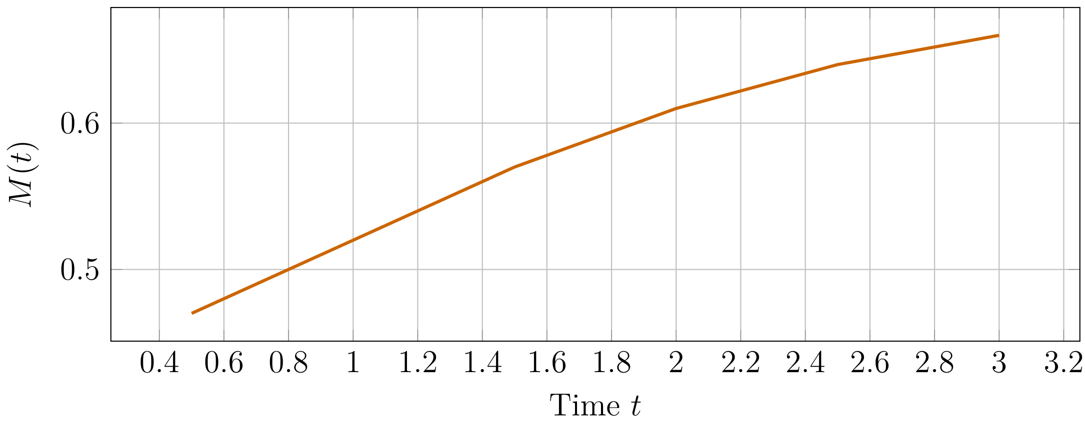

Figure 4: Moral observable $M(t)$ under $\Phi(M) = \frac{M}{1+\kappa M}$ (qualitative trajectory).

Annex F: Genealogical Continuity with Previous Work

This annex clarifies the relation between the present model and the earlier *Modification of the Model of Emergence of Morality.*[48]

Continuities

- The base differential system $\dot{x} = x_1 y^2 - x_2 x$, $\dot{y} = B_1 x - B_2 y$ is preserved.[49]
- The observable $M(t) = \sigma_1 x + \sigma_2 y$ continues to represent morality as a weighted sum of action and obligation.[50]

Extensions

- Introduction of ethical reflection $E(t)$ with its own dynamics and coupling.[51]
- Stability analysis via Jacobian and Routh–Hurwitz criteria.[52]
- Annexes with numerical examples, graphical simulations, and applied case studies.[53]

Interpretive note

The present paper should be read as a *second generation* of the emergence model: not a repetition, but a bifurcation into a richer system where morality and ethics interact

[48] J. Mas i Manjón, *Modification of the Model of Emergence of Morality: A Mathematical Model*, Academia.edu (2024).
[49] This ensures genealogical legitimacy: the emergent dynamics of morality remain anchored in the original formulation.
[50] Weights σ_1, σ_2 retain their interpretive role as calibration parameters.
[51] This annex highlights the novelty: ethics is no longer external commentary but an endogenous variable.
[52] The previous work hinted at stability; here it is formalized with full criteria.
[53] This expansion shows practical resonance in politics, economy, technology, and culture.

dynamically.[54]

References

This annex consolidates all references cited throughout the paper, ensuring genealogical continuity and scholarly legitimacy.[55]

Primary Works

- Mas i Manjón, J. (2024). *Modification of the Model of Emergence of Morality: A Mathematical Model*. Academia.edu. Available at https://www.academia.edu/120720065/Modification_of_the_Model_of_Emergence_of_Morality_a_mathematical_model.
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- Ricoeur, P. (1990). *Soi-même comme un autre*. Paris: Seuil.
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- Picard, É. (1890). *Sur lexistence des intégrales des équations différentielles*. Journal de Mathématiques Pures et Appliquées.
- Routh, E.J. (1877). *A Treatise on the Stability of a Given State of Motion*. London: Macmillan.
- Hurwitz, A. (1895). *Über die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reellen Teilen besitzt*. Mathematische Annalen.

[54] This genealogical annex prevents confusion: readers familiar with the earlier work will recognize continuity yet appreciate the new dimensions.

[55] The annex serves as a safeguard against fragmentation: all sources are listed here, even if already included in the BibTeX file.

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@articleMas2024, author = Jordi Mas i Manjón, title = Modification of the Model of Emergence of Morality: A Mathematical Model, year = 2024, journal = Academia.edu, url = https://www.academia.edu/120720065/Modification$_{o}f_{t}he_{M}odel_{o}f_{E}mergence_{o}f_{M}orality_{am}athematica$

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@bookRawls1993, author = John Rawls, title = Political Liberalism, year = 1993, publisher = Columbia University Press

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